ETH zürich

AI Center Projects
in Machine Learning

Causal Discovery via Conditional Divergence

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1 Introduction

Distinguishing cause and effect is a fundamental problem in science. The most elementary bivariate case, however, is already highly challenging using solely observational data [5].
Formally, given two random variables X and Y, we want to distinguish between three cases:

1. X causes Y (noted $X \to Y$).
2. $X \leftarrow Y$.
3. X and Y are not causally related (either because they are independent or because of confounding phenomena).

The central actor of our analysis is CDCI [1], a novel algorithm that achieves state-of-the-art performance. Our contributions include the proposal of two extensions, and an empirical performance analysis in the presence of hidden confounders. We release an open source package available on PyPI [4].

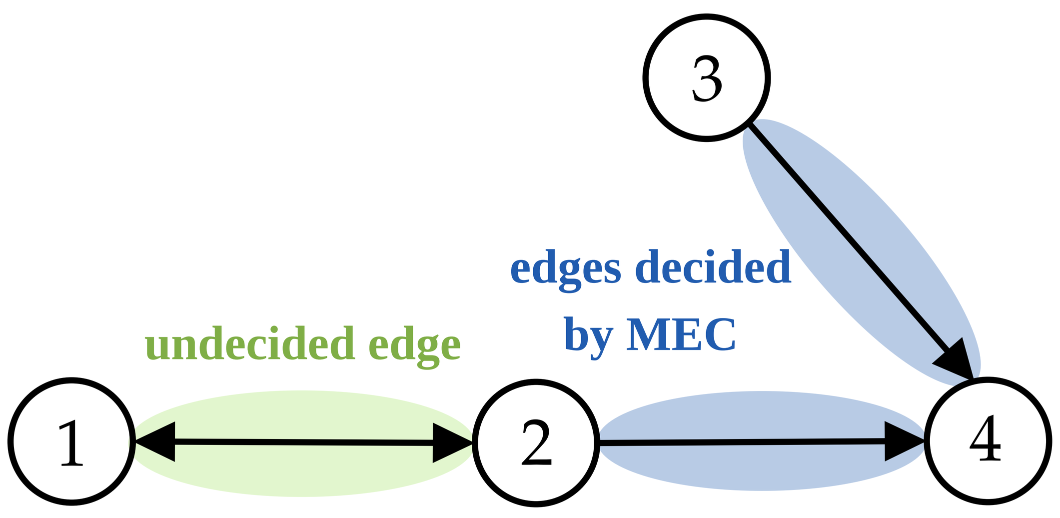

Figure 1: An example graph with one undecided edge in the MEC. CDCI can help distinguish the edge direction based on conditional divergence of variables.

2 Method Overview

CDCI relies on the fundamental assumption that if $X \to Y$, then the conditional distribution $Pr[Y|X=x]$ is invariant for different values of x [2, 3, 6]. Define the *Normalized Conditional Divergence* (NCD):

$$NCD_D(Y|X) := \mathbb{E}_{x\sim P[X]} \quad [D(P[\hat{Y}|X=x], P[\hat{Y}])] \tag{1}$$

where D is a valid probability distance measure and $\hat{Y}$ is a standardized conditional distribution of Y given $X = x$.

Algorithm 1 The Conditional Divergence based Causal Inference (CDCI) algorithm for causal direction prediction based on conditional distribution divergence.
Input: An i.i.d. bivariate joint distribution $P(X, Y)$, and a probability distance D.
Output: The causal score $C_{X\to Y}$ and direction. 1. Compute the conditional divergences for both directions using the probability distance D, as in Equation 1:

$$C_{X|Y} := NCD(X \mid Y) \text{ and } C_{Y|X} := NCD(Y \mid X)$$

2. Compute the causal score:

$$C_{X\to Y} := C_{X|Y} - C_{Y|X}$$

3. Output the causal score $C_{X\to Y}$ and

$$\text{direction} := \begin{cases} X \to Y & \text{if } C_{X\to Y} > 0 \\ Y \to X & \text{if } C_{X\to Y} < 0 \\ \text{Non-causal} & \text{if } C_{X\to Y} = 0 \end{cases}$$

3 Project Scope & Goals

- **Reproduction of results and code**
  - Simple implementation & reproducible ✓
- **Extension to novel divergence measures**
  - Jensen-Shannon (JS), Maximum-Mean (MMD), Wasserstein (+ others) ✓
- **Refuting claim: Distinguish between confounding and causation**
  - Counter Example (see Fig. 3) ✓
- **Ablation study: rationale of binning the conditionals** ✓

4 Results & Discussion

Novel Divergence Measures

AUROC of divergence measures for different datasets

	Average	CE-Gauss	CE-Multi	CE-Net	D4S1	D4S2A	D4S2B	D4S2C
CCS	0.744	0.905	0.960	0.897	0.613	0.685	0.566	0.584
CHD	0.750	0.914	0.976	0.905	0.627	0.676	0.580	0.574
CKL	0.744	0.910	0.955	0.893	0.604	0.687	0.575	0.584
JS (ours)	0.751	0.916	0.976	0.912	0.609	0.682	0.583	0.578
MMD LAP (ours)	0.755	0.918	0.959	0.919	0.609	0.697	0.582	0.603
WASS (ours)	0.722	0.891	0.911	0.920	0.618	0.621	0.534	0.558

Figure 2: Performance of multiple divergence measures for CDCI on different datasets.

Confounding Experiment

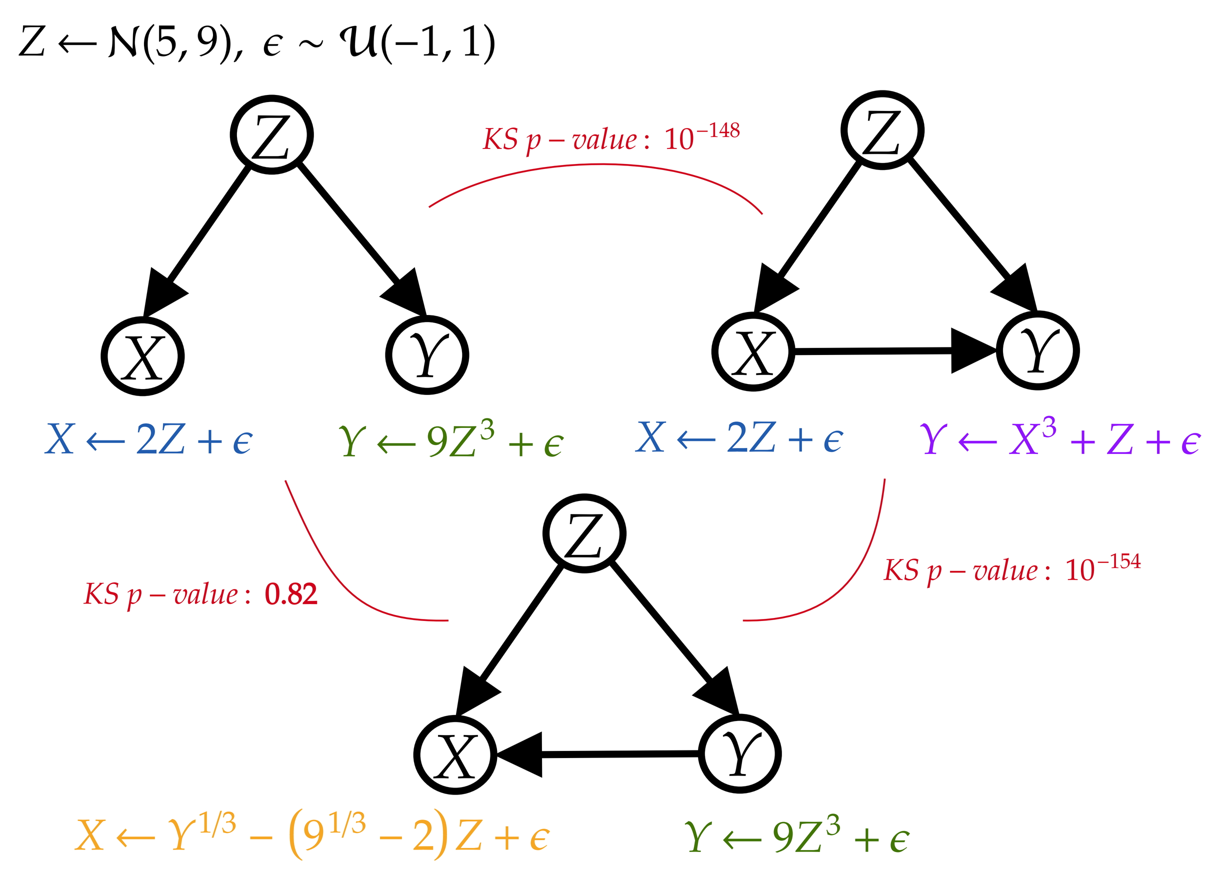

Figure 3: Different confounding sets show that CDCDI cannot, as claimed, distinguish confounding from causation.

5 Conclusion

Strong points:

- No reliance on strong restricting assumptions
- Simple idea & implementation
- Nonetheless good performance

Weak points:

- No verified or provable theory
- False claims about confounding

Contributions: SM: implemented new divergence measures, PyPi package and investigated hidden confounder case; YO: performed kernel search for MMD, did ablation study with diff. binning strategies; AH: made plots and illustrations and prepared poster; all 3 were involved in reproduction of code and results.

References

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